CSC4010: Introduction to Theoretical Computer Science
Problem Set 3

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1. Consider the language $L = \{\langle M \rangle \mid L(M) = \overline{A}_{TM}\}$. Is this language decidable or undecidable? Justify your answer by giving a decider if it is decidable, otherwise formally prove it is undecidable (via one of the methods described in class).

2. Consider the language $L = \{\langle M \rangle \mid M \text{ is a Turing machine and } L(M) \text{ is decidable}\}$. Is this language decidable or undecidable? Justify your answer by giving a decider if it is decidable, otherwise formally prove that it is undecidable (via one of the methods described in class).

3. Rice's Theorem states that it is impossible to decide any non-trivial property of recognizable languages. More concretely, a property P is a subset of recognizable languages, and it is said to be non-trivial if there exist a Turing Machine M_1 such that $L(M_1) \in P$, and there exists M_2 such that $L(M_2) \notin P$. Let $L_P = \{\langle M \rangle \mid L(M) \in P\}$. Show that it is undecidable.

   For each language L below, determine whether it is decidable. If it is undecidable, prove it. In your proof, clearly state whether you are using Rice's Theorem or a direct reduction. If you use Rice's Theorem, explain how the property you use is non-trivial. If you do not use Rice's Theorem, explain why it does not apply.

   (a) The language $L = \{\langle M \rangle \mid L(M) \text{ is recognizable}\}$.

   (b) The language $L = \{\langle M \rangle \mid M \text{ has at most 30 states}\}$.

   (c) The language $L = \{\langle M \rangle \mid M \text{ halts on } \epsilon\}$.

   (d) The language $L = \{\langle M \rangle \mid L(M) \text{ recognized by some Turing machine having an even number of states}\}$.

   (e) The language $L = \{\langle M \rangle \mid L(M) \text{ is recognized by a TM with at most 30 states and a tape alphabet with at most 30 symbols}\}$.

4. Consider the language $L = \{\langle M \rangle \mid L(M) \text{ contains a palindrome}\}$. Is this language decidable or undecidable? Is it recognizable or unrecognizable? Justify your answer.

5. Consider the language $L = \{\langle M \rangle \mid M \text{ accepts string } 01 \text{ in perfect-square number of steps}\}$. Is this language decidable or undecidable? Justify your answer.

6. Consider the language $L = \{\langle M \rangle \mid \forall w \in \{0,1\}^*, M \text{ accepts } w \text{ iff } M \text{ accepts } w^R\}$. Is this language decidable or undecidable? Justify your answer.

7. Consider the problem of determining whether a two-tape Turing machine ever writes a nonblank symbol on its second tape during the course of its computation on some input string. Formulate this problem as a language and show that it is undecidable.