CSC4010 (Practice Midterm) **Date: 22 October 2026**
Name:

Instructions: This exam is closed book, closed notebook. No electronic devices are allowed. You may use one A4-size study sheet (one-sided) that you prepared yourself. The last few pages of the exam are scratch paper; you may not use your own. **Please print your ID number at the top of each page.**

Any deviation from these rules may lead to cancellation of your exam.

The exam consists of 3 multiple-choice, 4 short-answer, and 2 longer-answer questions, for a total of 36 points. For the short- and longer-answer sections, please write your answers clearly in the spaces provided. If you run out of room or need to start over, you may use the blank pages at the end, but you MUST make that clear in the space provided for the answer. The exam has 4 pages printed on both sides, including this page and the blank pages at the end.

Multiple Choice: Select <u>all</u> valid options (4 points each)

For each of the problems in this section, select <u>all</u> valid options; this could be all of them, none of them, or something in between.

A fully correct solution earns all 4 points. Each error (selection or non-selection) reduces your score by 1 points (to a minimum of 0 points).

1. (4 points) Select all of the **countable** sets.

   ☐ $\{\langle M\rangle : M \text{ is a Turing Machine}\}$
   ☐ $\{\langle M\rangle x : M \text{ is a Turing Machine and accepts } x\}$
   ☐ $\{L : L \subseteq \{0,1\}^*\}$
   ☐ $\{L : L \subseteq \{0,1\}^* \text{ is a finite language}\}$

2. (4 points) Select each hypothesis that implies that L is decidable.

   ☐ Language $\bar{L}$ is decidable.
   ☐ Language A is decidable and $L \subseteq A$.
   ☐ Language $A \leq_m L$ and A is decidable.
   ☐ Language $L = A \cup B$, and at least one of A or B is decidable.

3. (4 points) Consider three algorithms with the following worst-case running times:

$$T_A(n) = O(n^3)$$
$$T_B(n) = \Theta(n^2)$$
$$T_C(n) = \Theta(n \log n)$$

   Recall that the worst-case running time represents the maximum number of steps an algorithm takes over all inputs of size n. **Select all statements that are necessarily true based on the given information.**

   ☐ Algorithm A has worst-case running time $\Omega(n^2)$
   ☐ Algorithm B has worst-case running time $O(n^3)$
   ☐ Algorithm C runs faster than Algorithm B for all inputs.
   ☐ For all large enough n, there exists an input of size n for which C runs faster than B.

Short Answer (3 points each)

1. (3 points) Consider the set of all bijections $f : \mathbb{N} \to \mathbb{N}$. Is this set countable or uncountable? Give a short explanation.

2. (3 points) Consider a variant of a Turing machine with a two-way infinite tape. At each step, its head reads and writes only the current cell and then moves exactly two cells to the left or right, without reading or writing the skipped cell. The binary input is written in consecutive cells, and the head initially points to the first input cell. Can this variant decide every decidable language over $\{0, 1\}$? Give a short explanation.

3. (3 points) Let $A, B \subseteq \{0,1\}^*$ be recognizable languages. Show that $A \cup B$ is recognizable.

4. (3 points) Let $\mathrm{COMPOSITE}$ be the language of nonempty binary strings representing composite integers. Briefly describe a polynomial-size certificate and a polynomial-time verifier showing that $\mathrm{COMPOSITE} \in \mathrm{NP}$.

Long Answer Question (6 points each)

1. (6 points) Let $L_{\text{even-acc}} = \{\langle M \rangle \mid \forall x \in \{0,1\}^*,\ M(x) \text{ accepts } \iff |x| \text{ is even}\}$. That is, let $L_{\text{even-acc}}$ be the language of descriptions of all Turing machines that accept exactly the set of even-length strings. Show that this language is unrecognizable.

2. (6 points) Consider the language $L = \{\langle M \rangle \mid L(M)$ contains a string of the form $0^n 1^n$ for $n \geq 0\}$. Is this language decidable or undecidable? Is it recognizable or unrecognizable? Justify your answer.